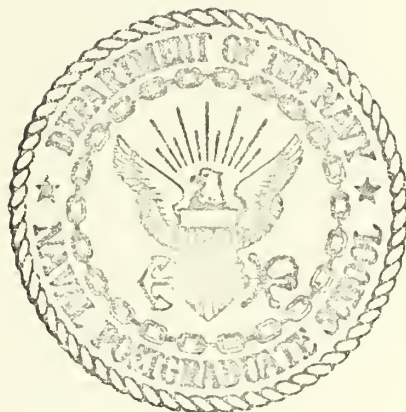


A LANCHESTER-TYPE MODEL WITH
LOGISTICS CONSIDERATIONS

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THESIS

A LANCHESTER-TYPE MODEL WITH
LOGISTICS CONSIDERATIONS

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September 1973

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Logistics Considerations

by

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ABSTRACT

In this thesis a Lanchester-Type model of combat with logistics considerations is presented. The combat effectiveness of each force is related to its supply. Four basic groups of force supplies are considered: food (all goods used whether or not combat is in progress); ammo (goods used only in combat activity); fuel (goods required for mobility); and capital goods, which are used to increase or replace the capacity of the logistics pipeline. Lanchester attrition-rate coefficients are considered to be functions of the level of food and ammo supplies.

In the model, each opponent has a main battle force, a reserve force, a logistics pipeline defense force, and a force which may attack the other side's logistic pipeline. Differential equations for the combat dynamics are derived, and some possible objectives and battle termination conditions are suggested.

An example of use of the model is given, and some analytical techniques for studying the model are discussed. Related topics for further research are suggested.

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I. INTRODUCTION

A. BACKGROUND ON LANCHESTER-TYPE MODELS OF WARFARE

During World War I a British aeronautical engineer, F. W. Lanchester, postulated a differential equation model of combat. His reason for doing this was to explain the principle of concentration of firepower. Using his model one can study and analyze mathematically the process of combat attrition. Others have subsequently expanded and developed Lanchester's equations into what has become known as Lanchester-Type models of combat. Essentially, a Lanchester-Type model of combat is a set of differential equations which describe mathematically the interactions of opposing combat forces. When this set of equations is solved for force levels as a function of time, the conditions necessary for one force to win (given a definition of winning, such as driving the opposing force level to zero) may be obtained.

Lanchester-Type models are deterministic in the sense that given a set of initial conditions, the winning force is known with certainty. This is in contrast with a stochastic model in which given the same set of initial conditions, a probability of winning may be determined for each force; i.e., the outcome (winner) is not known with certainty. The model developed in this thesis is a deterministic model.

The usefulness of a Lanchester-Type model is that such a model can give some insight into the over-all dynamics of a combat situation. Using a Lanchester model, one may learn,

for example, which of a set of possible tactics appear to be "better" in a given situation. "Better" could be thought of in terms of winning a battle in a shorter period of time, or winning a battle while suffering less casualties. One may also learn why a particular tactic is successful, by studying the mathematical formulation of the combat dynamics. In this light, a simple Lanchester model is much more useful than a detailed simulation model of combat, in which the major cause-and-effect relationships may not be readily apparent to the user. In addition, a Lanchester model is generally more responsive; that is, numerical answers are obtained usually with significantly less time and effort than with a detailed simulation model.

B. INCORPORATION OF LOGISTICS CONSIDERATIONS INTO LANCHESTER FORMULATIONS

It has long been known that logistics, or supply lines, or lines of communication, are vital to a successful military campaign strategy. Since the time of Mahan with his writings on sea power, [Ref. 1] attempts have been made to qualitatively assess the value and importance of the ability to provide support to combat forces. It has been only recently, however, that any attempt to model, in a quantitative and analytic way, the logistics element of combat has been made. Moglewer and Payne [Ref. 2] have studied the problem of two forces, one of which has a supply pipeline, as a two-person zero-sum differential game. Some ideas presented in their paper are expanded upon in this thesis. The model developed here is broader in scope than that of Moglewer and Payne.[Ref. 2]

II. ANALYSIS OF LOGISTICS EFFECTS ON FORCE CAPABILITIES

A. LANCHESTER'S LINEAR LAW, SQUARE LAW AND ATTRITION-RATE COEFFICIENTS

From classical Lanchester theory, two "laws" have evolved: the "square law" and "linear law" of combat attrition.

The "square law" is derived from the following scenario. Suppose two forces, X and Y, are engaged in combat with each other. Let $X(t)$ represent the number of men in the X force at time t , and $Y(t)$ represent the number of men in the Y force at time t . Assume that the rate at which the X force inflicts casualties on the Y force is proportional to the number of men in the X force; and that a similar attrition process occurs for the X force which is proportional to the number of men in the Y force. The constants of proportionality need not be the same. Mathematically, the rate of change of the X force is equal, in magnitude, to the number of Y at time (t) times the constant of proportionality. Since $Y(t)$ is non-negative, and the constant of proportionality is, by convention, positive, the right hand side of the equation carries a minus sign, because $X(t)$ is decreasing with time (i.e., $\frac{dX(t)}{dt} < 0$).

Thus one has that:

$$\frac{dX(t)}{dt} = -aY(t)$$

and

$$\frac{dY(t)}{dt} = -bX(t)$$

The above two equations describe the combat dynamics. If these equations are rewritten as

$$\frac{dX(t)}{dY(t)} = \frac{aY(t)}{bX(t)}$$

and if we let $X(t=0) = X_0$, and $Y(t=0) = Y_0$, we readily obtain $b(X_0^2 - X(t)^2) = a(Y_0^2 - Y(t)^2)$. The appearance of the force levels as square terms gives the label "square law".

If the above scenario is modified such that the rate of inflicting casualties is proportional to both X and Y, we get that:

$$\frac{dX(t)}{dt} = -aX(t)Y(t)$$

$$\text{and } \frac{dY(t)}{dt} = -bX(t)Y(t)$$

Rewriting these equations as before, and solving, we obtain $b(X_0 - X(t)) = a(Y_0 - Y(t))$. This is known as the linear law.

In the above two models, the constants of proportionality a and b are commonly known as attrition rate coefficients. These constants express the strength, or kill capability, of one force against the other. If a is larger than b, for example, it means that the Y force has more kill capability per man than the X force.

B. BATTLE TERMINATION CONDITIONS

The above models are often analyzed in terms of a "fight to the finish" which means the battle continues until either $X(t) = 0$ or $Y(t) = 0$ (or possibly both at the same time). The events $X(t) = 0$ and $Y(t) = 0$ are the battle termination conditions. There exist an infinite number of possible events from which one might choose a battle termination condition, or set of conditions. Few, if any, real battles result in one hundred percent casualties being suffered by either force. Thus, analysis of real situations might be accomplished using battle termination conditions such as $X(t) = 0.8 X_0$ or $Y(t) = 0.8 Y_0$; that is the battle continues until either X or Y suffers twenty percent casualties. The force which suffers twenty percent casualties then is said to lose the battle. Having chosen the battle termination conditions, one may determine conditions on X_0 , Y_0 , a , and b which guarantee that X or Y win.

C. FACTORS WHICH AFFECT THE LANCHESTER ATTRITION-RATE COEFFICIENTS

It was implicitly assumed in the above models that the attrition-rate coefficients a and b were constants. This assumption makes the differential equations easy to solve; however, there is no reason why the kill capability (which the attrition-rate coefficient represents) of a force should remain constant over the course of a battle. In fact, kill capability could depend on such factors as range, weather, visibility, discipline and morale, supply inventories, etc.

which vary with time. In order to study the effects of such factors on combat processes, one can extend the basic Lanchester models given above by considering variable attrition-rate coefficients. Solutions to these extended equations are, in general, much more complicated than those to constant coefficient equations. Moreover, it is the exception rather than the rule that such a solution can be expressed in terms of any tabulated functions.

Attempts to define, and quantify some of the factors affecting the attrition-rate coefficients exist in the open literature. In 1953, H. R. Weiss noted several deficiencies in original Lanchester theory [Ref. 3]. In 1957, he proposed some modifications to overcome some of these deficiencies, among which he included a range dependent attrition-rate coefficient [Ref. 4]. Brackney in 1959 addressed the effect of enemy force size on target acquisition times and showed that this dependence on force size can determine the attrition process (whether square law or linear law)[Ref. 5] However in the models studied above, it is usually assumed that neither force has any logistics limitations. This means infinite sources of ammunition are available if required to finish the battle. It is the purpose of this thesis to formulate a model in which logistics and supply levels are treated explicitly in a realistic, yet reasonably simple, manner. This is done by developing attrition-rate coefficients which are functions of supply levels. For simplicity, effects from other factors (such as those discussed above)

on attrition-rate coefficients will be ignored. In a generalized model of combat, effects from all of these factors should be considered.

These factors could be considered to act independently of each other, and therefore the generalized attrition-rate coefficient could be taken as a product of factors (each of which could itself be a function of input variables and time). For example, if the effects of discipline and morale were determined to be some function of force levels and time, say $a_d(X,Y,t)$, and the effects of supply levels were some other function, $a_s(X,Y,t)$ then the composite function $a(X,Y,t) = a_d(X,Y,t) \cdot a_s(X,Y,t)$ would be a product of the discipline function and supply function.

One might argue, however, that if supplies were drastically low, discipline could weaken as a result. In the model, however, if the low supply level were reflected in $a_s(X,Y,t)$ being close to or equal to zero, it would not matter what value $a_d(X,Y,t)$ were to have; $a(X,Y,t)$ would be at or close to zero anyway. In other words, a perfectly disciplined force cannot overcome a lack of ammunition available to use against an enemy, and thus a force with a weakened discipline could do no worse.

D. ANALYSIS OF LOGISTIC SUPPORT AND SUPPLY LEVELS

The idea that logistic support is vital to the success of military operations is, of course, not new. Karl von Clausewitz in his classical writings states that in modern wars, "arrangements for subsistence shall be on an adequate

scale."[Ref. 6] In 1807 Napoleon defined the secret of warfare as the art of maintaining one's own communications and gaining possession of the enemy's.[Ref. 7] The Russian general Denis Davidov in his essay on partisan war states this type of warfare (partisan) "is concerned with the entire area which separates the enemy from his operational base" and its objectives are "to cut the communications lines, destroy all units and wagons wanting to join up with him, inflict surprise blows on the enemy left without food and cartridges and at the same time block his retreat..."[Ref. 7] More well-known, especially in Navy circles, are the principles laid down in Mahan's writings on sea power. In particular, he said: "the most important of strategic lines are those which concern the communications. Communications dominate war. This has peculiar force on shore, because an army is immediately dependent on supplies frequently renewed. It can endure a brief interruption much less readily than a fleet can..."[Ref. 1]

In recent history, perhaps the best example of putting this principle of severing an army's communication lines is the United States forces landing at Inchon Korea in September of 1950. The purpose of this landing was to capture Seoul. Gen. MacArthur's argument for this operation was that seizure of Inchon and Seoul would cut the enemy's supply line and seal off the entire southern Korea Peninsula. The vulnerability of the enemy was his supply position. The enemy's supply lines from the north converged on Seoul; from

Seoul they radiated to the south. By seizing Seoul, the enemy's supply system would be completely paralyzed. This in turn would paralyze the fighting power of the troops. Without munitions and food they would soon be helpless, and disorganized, and could easily be over-powered by smaller, but well supplied troops.[Ref. 8] That the Inchon landing was successful, and the objective accomplished, is well known.

Thus, the situation in Korea is a clear example where application of Lanchester theory, without logistic considerations, would predict the enemy to be the winner simply because of his vast numerical superiority. Because logistics are such a vital factor in warfare, both in theory and in practice, it seems necessary that logistics considerations be included in any realistic applications of Lanchester-Type models of combat.

In order to incorporate logistic support into a mathematical model, an analysis of the concept of logistic support must be made. In the broadest sense, logistic support is the availability, transportation, storage and final distribution of all material goods for the combat forces. The availability of material goods, for the purposes of the model developed in this thesis, will be thought of as the basic industrial capacity or production capacity of goods in the homeland, and will be treated as unconstrained. The transportation, storage and final distribution of these goods from the homeland to the combat area will be called the pipeline. The total amount of goods which can be delivered

to the combat forces over a given period of time is bounded by the pipeline capacity.

Numerous types of goods and supplies are required to support the modern combat force. Army field manual FM - 101 - 10 - 1 breaks logistic support down into ten classes, each class having sub-classifications. The supply classes and elements of each are presented in tables. This classification system is designed for the peacetime as well as wartime army, however, and therefore includes many items which are irrelevant for purposes of a model of combat.

Certain supplies are necessary for the day to day operations of forces, independent of whether combat is in progress or not. Among these are food; administrative support items; fuel for power generators, non-combat transportation, stoves; etc; clothing; and maintenance items. For convenience, this classification of supplies which are used independent of the combat situation will be labeled "food".

Another group of supplies is required only during combat. Items included are ammunition; fuel for weapons systems such as tanks and aircraft; medical supplies; and replacement parts and equipment such as rifles, tubes for artillery weapons, etc. This group of supplies required only for combat operations will be called "ammo".

A third class of material goods is necessary for providing mobility to the forces, whether or not combat is in progress. These supplies are consumed only in advancing or retreating; and do not include supplies used in the local

transportation requirements of a stationary force whether engaged in combat or not. The predominant element in this class is, of course, motor fuel. Accordingly, this classification is termed "fuel."

The fourth and last group of goods to be included in this model is that of investment-type goods. This type of material is used to expand or replace the pipeline capacity of the force. Included would be such items as building material, building equipment, transportation equipment, and materials handling equipment. Supplies required for the day-to-day operation of the pipeline are not included here; they are included in "food". This group is composed primarily of capital goods; thus it will be called "capital".

In summary, let us consider all supplies to be in four groups:

Food: (all items consumed independent of combat and/or mobility.)

Ammo: (all items consumed only by engaging in combat)

Fuel: (items consumed only by engaging in a major movement such as complete force withdrawal or advance)

Capital: (items used to increase or replace the pipeline capacity)

Since mobility will not be a factor in the model developed in this thesis, the group labelled fuel will not be considered further.

E. COMBAT EFFECTIVENESS AS A FUNCTION OF SUPPLY LEVELS

In order to develop functional expressions for combat effectiveness in terms of supply levels, the utilization of

each of the groups of supplies must be considered. That is, the combat effectiveness with no supply, "normal" supply, and "over supply", as well as the trend of combat effectiveness with increasing supply levels at each of these supply levels (zero, normal, over supply) should be ascertained. Precise determination of the effectiveness in terms of supply levels is a separate operations research problem in itself. In at least one model of combat [Ref. 9] combat effectiveness is calculated as an index of firepower. This index is tabulated for different levels of supply.

For the model developed in this thesis, a continuous function of combat effectiveness in terms of supply level is developed from the current doctrine contained in the U.S. Army Field Manual 101-10-1. Paragraph 5.9 of this manual, titled "ammunition Supply Levels" is quoted in part: "levels of ammunition supply are normally expressed as days of supply for each theater of operations. Available ammunition assets then are announced as available supply rates (ASR); as tactical experience is gained, the required supply rate (RSR) is computed. The ammunition logistics supply system relates ASR to RSR so that a desirable balance of reserves is maintained. Major commanders will desire to maintain a planned level of supply as the minimum level of reserves. To change this level of supply by increasing the level of receipts requires a planning leadtime that may be in excess of tactical and operational planning leadtimes."

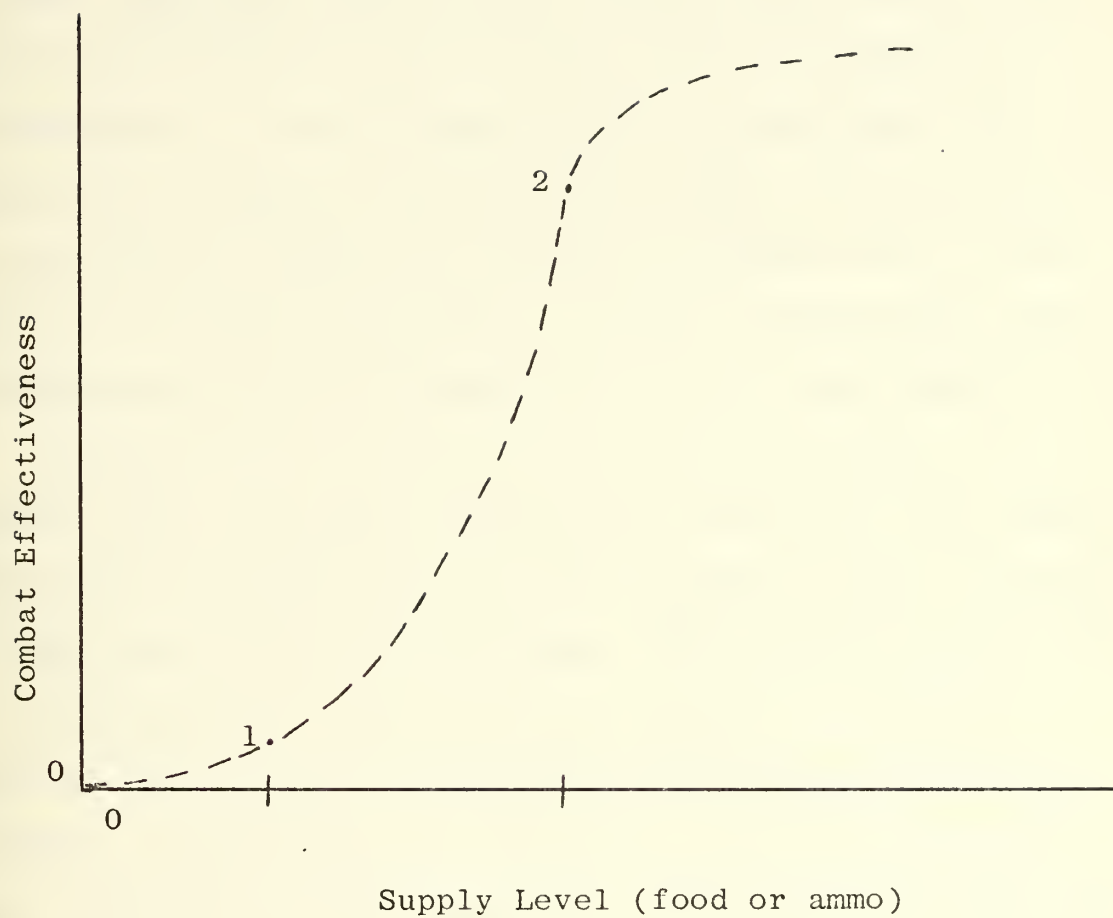
Similar reasoning could be applied to food supply levels. Certainly it is desirable to maintain some minimum level of reserves of food simply in order to preserve morale and discipline.

From the above discussion, the following assumptions can be made concerning the effect of ammunition supplies on combat effectiveness.

- A1. A supply level of zero implies zero effectiveness.
- A2. As supplies increase toward the minimum reserve level, effectiveness increases slowly, because of the commanders unwillingness to expend ammo below this minimum level.
- A3. As supplies increase above the minimum supply level, effectiveness will increase rapidly.
- A4. As supplies continue to increase toward over-supply, effectiveness will increase more and more slowly, because the large supplies may become almost a burden, and supplies become sufficient to maintain the maximum sustained firing rates.

Figure 1 depicts a curve of combat effectiveness versus supply level for ammo which was constructed from these assumptions.

Assumptions A2, A3 and A4 are reasonable to apply to food supplies. The applicability of assumption A1 to this model would depend on the proposed use of the model. For example, to model a campaign which took place over several weeks or months, obviously a food supply level of zero would



1. Minimum level of supplies kept in reserve.
2. Normal (full allowance) level of supplies

Figure 1

reduce military effectiveness to zero or at least a very low level. If the scenario were that of a battle whose duration would be measured in days or hours, the effect of no food supplies would not be nearly so drastic. In such a case, assumption A1 might have to be modified; however in a battle of short duration, combat effectiveness would probably not depend very heavily on food supply levels as compared to its dependence on ammo supply levels; in that case food supply levels could be ignored in the model. For purposes of this thesis, therefore, it will be assumed that if food supply levels are significant to combat effectiveness, the relationship will be the same as for ammo. (see figure 1)

From the above discussion, and from the figure, it is apparent that the general form of the curve is an "S" shape. There are many functions which can be made to take the form of this curve; as noted previously, the precise determination of the functional relationship is a separate problem in itself. For purposes of simplicity and mathematical convenience, a function of the form $f(s) = K(1 - e^{-as})$ will be used, where $f(s)$ is the Lanchester attrition-rate coefficient, s is supply level, a is a shape parameter which is determined by the sensitivity of effectiveness to changes in supply levels. The k is constant determined by maximum effectiveness. This function has the following useful properties:

- 1) $f(0) = 0$
- 2) $\lim_{s \rightarrow \infty} f(s) = K$ i.e., effectiveness as a function of supply level cannot increase above some upper bound.

3) It is both continuous and differentiable.

Investment supplies have no direct influence on combat effectiveness. The only purpose for considering this group of supplies in this model is that such goods are required to increase the capacity of the logistics pipeline, or to replace capacity lost by damage from the enemy. In order to obtain these investment goods in the theater of operations, however, part of the existing pipeline capacity must be utilized, thereby decreasing the available capacity for food and ammo. This reduced capacity will then have an effect on supply levels of food and ammo, and therefore on combat effectiveness. This effect will be discussed in more detail in the steady state analysis of the model.

III. FORMULATION OF THE MODEL WITH LOGISTICS EFFECTS

A. MODEL SCENARIO

Two major combatants, X and Y, are engaged in a campaign. Each combatant has a main battle force, a reserve force, and a special force which can attack the other combatant's logistics pipeline, and a pipeline defense force. Pipeline defenders may engage, and be engaged by, pipeline attackers. Pipeline terminations are at the location of the respective combatants reserve force location, which is a secure area. Thus supplies which are delivered through the pipeline are subject only to use but not to attack or enemy damage. Each force (main battle, pipeline attackers and pipeline defenders) may be reinforced by reserves at rates which have a maximum value; similarly each of the forces may withdraw to the reserve position at rates which have a maximum value. All forces consume food at the same rate; ammo is consumed only by the main battle and pipeline attackers and defenders. The objective of each combatant commander is to annihilate the other combatant or reduce its effectiveness to zero. Decisions available to each commander are force allocations and supply allocation. Force allocations decisions are made by reinforcing or withdrawing troops in combat; pipeline allocations are made by deciding the proportion of pipeline capacity to be utilized for each of the supply groups: food, ammo and investment. A graphic representation of this is depicted in figure 2.

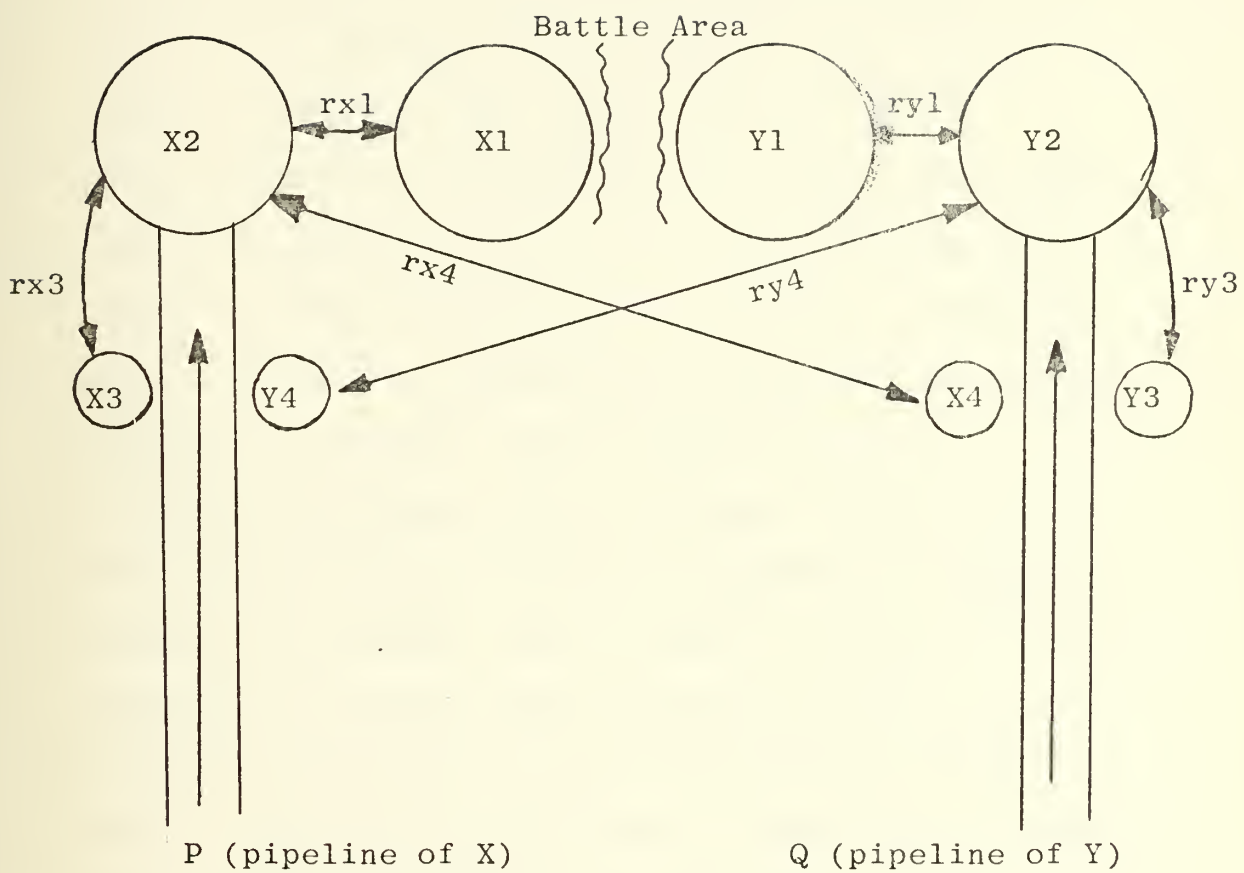


Figure 2

B. DERIVATION OF DIFFERENTIAL EQUATIONS

1. List of Variables and Coefficients

The differential equations describing the dynamics of the model are developed below. The following list of variables and coefficients is presented for clarity:

$X_1(t)$ = main battle force level of X at time t

$X_2(t)$ = reserve force level of X at time t

$X_3(t)$ = pipeline defense force level of X at time t

$X_4(t)$ = pipeline attack force level of X at time t

$Y_1(t)$ = main battle force level of Y at time t

$Y_2(t)$ = reserve force level of Y at time t

$Y_3(t)$ = pipeline defense force level of Y at time t

$Y_4(t)$ = pipeline attack force level of Y at time t

$P(t)$ = pipeline capacity of X at time t (tons/day)

$Q(t)$ = pipeline capacity of Y at time t (tons/day)

$S_{xa}(t)$ = X's supply level of ammo at time t

$S_{xf}(t)$ = X's supply level of food at time t

$S_{xc}(t)$ = X's supply level of investment at time t

$S_{ya}(t)$ = Y's supply level of ammo at time t

$S_{yf}(t)$ = Y's supply level of food at time t

$S_{yc}(t)$ = Y's supply level of investment at time t

C_{xa} = X's consumption rate of ammo when in combat

C_{xf} = X's consumption rate of food

C_{ya} = Y's consumption rate of ammo when in combat

C_{yf} = Y's consumption rate of food

$pxf(t)$ = fraction of X's pipeline capacity allocated for food

$\rho_{xa}(t)$ = fraction of X's pipeline capacity allocated for
ammo

$\rho_{xc}(t)$ = fraction of X's pipeline capacity allocated for
investment

$\rho_{xf}(t)$ = fraction of Y's pipeline capacity allocated for
food

$\rho_{ya}(t)$ = fraction of Y's pipeline capacity allocated for
ammo

$\rho_{yc}(t)$ = fraction of Y's pipeline capacity allocated for
investment

rx_1 = rate of reinforcing main battle force by X

ry_1 = rate of reinforcing main battle force at Y

rx_3 = rate of reinforcing pipeline defense force of X

ry_3 = rate of reinforcing pipeline defense force of Y

rx_4 = rate of reinforcing pipeline attack force of X

ry_4 = rate of reinforcing pipeline attack force of Y

2. Differential Equations for the Combat Dynamics

In this section differential equations are developed for the rate of change of each of the force and supply levels. A Lanchester "square-law" attrition process (i.e. attrition-rate proportional to the number of firers) is assumed for each force type.

Main Battle Force:

$$\frac{dX_1(t)}{dt} = -K(1-e^{-\alpha S_{ya}(t)})(1-e^{-\alpha S_{yf}(t)})Y_1(t) + rx_1(t)$$

This equation states that the time rate of change of the X main battle force is proportional to the Y main battle

force and to the rate of reinforcement, $rx1(t)$. The function of proportionality, $K(1-e^{-\alpha S_{ya}(t)})(1-e^{-\alpha S_{yf}(t)})$, is composed of the combat effectiveness function of Y's ammo and food supplies, and the factor K, which stands for any other combat effectiveness functions which might be included in a more general model. The constant α is a shape parameter. Both K and α are taken as numerically equal to one in the remainder of the discussion. Thus simplified, the equation for main battle force becomes:

$$\frac{dX1(t)}{dt} = - (1-e^{-S_{xa}(t)})(1-e^{-S_{xf}(t)})Y(t) + rx1(t).$$

A similar equation holds for the Y1 force.

Reserve Forces

$$\frac{dX2(t)}{dt} = -rx1 - rx3 - rx4$$

For $X2=0$, $rx1 + rx3 + rx4 \leq 0$.

$$\frac{dY2(t)}{dt} = -ry1 - ry3 - ry4$$

For $Y2=0$, $ry1 + ry3 + ry4 \leq 0$.

Reserve Forces are depleted at a rate equal to the algebraic sum of the reinforcement rates to main battle, pipeline attack and pipeline defense forces. If the reserve force is totally depleted, transfers between the fighting forces may be made, or forces may be withdrawn from the

fighting forces to the reserve force. Recall that rx1, rx2, etc. may be positive or negative numbers.

Pipeline Defense Forces:

$$\frac{dX3(t)}{dt} = -(1-e^{-S_{ya}(t)})(1-e^{-S_{yf}(t)})Y4(t) + rx3(t) \quad (X3>0)$$

$$\text{if } X3 \leq 0 \quad \frac{dX3(t)}{dt} = rx3(t)$$

$$\frac{dY3(t)}{dt} = -(1-e^{-S_{xa}(t)})(1-e^{-S_{xf}(t)})X4(t) + ry3(t) \quad (Y3>0)$$

$$\text{if } Y3 \leq 0 \quad \frac{dY3}{dt} = ry3(t)$$

Pipeline Attack Forces:

$$\frac{dX4(t)}{dt} = -(1-e^{-S_{ya}(t)})(1-e^{-S_{yf}(t)})Y3(t) + rx4(t) \quad (X4>0)$$

$$\text{if } X4 \leq 0 \quad \frac{dX4(t)}{dt} = rx4(t)$$

$$\frac{dY4(t)}{dt} = -(1-e^{-S_{xa}(t)})(1-e^{-S_{xf}(t)})X3(t) + ry4(t) \quad (Y4>0)$$

$$\text{if } Y4 \leq 0 \quad \frac{dY4(t)}{dt} = ry4(t)$$

Pipeline Dynamics:

$$\frac{dP(t)}{dt} = -ay(Y4(t)-X3(t)) + \rho xcP(t); (Y4(t) \geq X3(t))$$

$$\frac{dP(t)}{dt} = \rho xcP(t) \quad (Y4(t) < X3(t))$$

$$\frac{dQ(t)}{dt} = -ax(X4(t)-Y3(t)) + \rho_{yc}Q(t); (X4(t) \geq Y3(t))$$

$$\frac{dQ(t)}{dt} = \rho_{yc}Q(t) \quad (X4(t) < Y3(t))$$

These equations state that the pipeline capacity is destroyed at a rate proportional to the net amount of force deployed against it, and is rebuilt at a rate proportional to the amount of capacity allocated for capital goods. When the pipeline is defended more heavily than it is attacked, it is assumed that no capacity attrition occurs.

Supply Level Dynamics:

$$\frac{dS_{xa}(t)}{dt} = -C_{xa}(X1(t) + X3(t) + X4(t)) + \rho_{xa}P(t)$$

$$\frac{dS_{ya}(t)}{dt} = -C_{ya}(Y1(t) + Y3(t) + Y4(t)) + \rho_{ya}Q(t)$$

$$\frac{dS_{xf}(t)}{dt} = -C_{xf}(X(t)) + \rho_{xf}P(t)$$

$$\frac{dS_{yf}(t)}{dt} = -C_{yf}(Y(t)) + \rho_{yf}Q(t),$$

where $X(t) = X1(t) + X2(t) + X3(t) + X4(t)$; similarly for $Y(t)$.

The foregoing differential equations describe mathematically the combat and logistics dynamics of the model scenario. In these equations, the variables $rx1$, $rx3$, $rx4$, ρ_{xa} , ρ_{xf} , ρ_{xc} , and $ry1$, $ry3$, $ry4$, ρ_{ya} , ρ_{yf} , ρ_{yc} are variables which can

be controlled by the X and Y force commanders respectively. Through these variables, which are reinforcement (or withdrawal) rates and pipeline capacity allocations, the force commanders inject command decisions into the model. Thus these variables are called decision variables.

3. Objective Function

The objective function is a mathematical expression of the payoff, or results, of a battle or single play through of the model. The objective function should be a mathematical expression of the objective of the combat. From the standpoint of the X commander, for example, the survivors of the Y forces would be of negative value.

Care must be taken in the construction of the objective function to prevent an unrealistic situation from developing in the model. For example, from inspection of Figure 2 and the combat dynamics differential equations, it is evident that a possible path of evolution of the scenario is for one or the other, or both, commanders to direct all of his forces to the pipeline attack and defense forces; leaving no reserves and no main battle force. This is an unrealistic event in most combat situations. To prevent this outcome from occurring, the objective function must have the characteristic of forcing the force commanders to maintain as large main battle force as possible, on his own side, and minimize that of the enemy. This is done by assigning a positive value to X_1 survivors at the end of the battle, and a negative value to the Y_1 survivors, in the objective function to be maximized.

If desired and if appropriate in the scenario being analyzed, a value may be placed on the reserve forces also. When the objective is to win the main battle, the forces used for pipeline defense and attack will be allocated by the force commander to his best advantage such that he can win the main battle. Therefore no pipeline defense or attack forces are included in the payoff function. It is considered unrealistic in most cases to assign a payoff for large supply levels existing at the end of the battle. The supplies are used as tools for attaining the objective, but are not objectives in themselves. Thus supply levels are not included in the objective function presented here.

For the X commander, then, a possible expression for his objective is for him to maximize $X_1(T)$ and minimize $Y_1(T)$, where T denotes the time of the end of battle. T may be given or may be determined from the model itself. Formally, the X commander will:

Maximize $(M_1X_1(T)+M_2X_2(T)-N_1Y_1(T)-N_2Y_2(T))$
 $rx_i, \rho x_j$

$i = 1, 3, 4; j = a, f, c$

Subject to: (1) combat dynamics

(2) stopping rule

M_1, M_2 and N_1, N_2 are value coefficients assigned to the X and the Y survivors respectively.

4. . Battle Termination Conditions

The battle termination conditions, which will be called a "stopping rule" are conditions which when reached

mean the battle is over. If one of the forces becomes annihilated, for example, the battle is over. However, so far in the model, there are no restrictions on the values of the main battle force levels; that is, there is nothing to prevent $X_1(t)$ from taking on negative values. The model could continue "running" beyond the point where realistically the battle should stop. In order to prevent this, some realistic conditions are specified which, if met, stop the problem.

If either force depletes either ammo or food supplies, the battle should end. If either main battle force is annihilated, the battle is terminated. If the use of "breakpoints" is desired in the model, such as a maximum number or percentage of casualties before surrender or withdrawal, the battle force stopping rule is easily modified. The reduction of a force pipeline capacity to zero is not a valid stopping rule; with sufficient supply inventory the possibility of the affected force still being able to win the battle should be considered.

One possible set of battle termination conditions is the following. The occurrence of one or more of these conditions will stop the battle.

- (1) $X_1 = 0$
- (2) $Y_1 = 0$
- (3) $S_{xf} = 0$
- (4) $S_{xa} = 0$
- (5) $S_{ya} = 0$
- (6) $S_{yf} = 0$

5. Decision Variables:

The decisions available to each force commander to achieve optimal results are: whether or not to reinforce or withdraw troops from each of the main battle force, pipeline defense force, and pipeline attack force, i.e., troop allocation; and how much of his available pipeline capacity is to be used for each group or class of supplies, i.e., logistics allocation. Each of the reinforcement rates may be selected from values between maximum (reinforcement) and minimum (withdrawal) rates. (Note that for example $rx1 < 0$ means X's main battle force is withdrawing to the reserve force). These limiting values are input to the model.

Logistics allocations are made by selecting the allocation fractions ρ_{xa} , ρ_{xf} , and ρ_{xc} such that each allocation fraction is non-negative, and the sum of the allocation fractions does not exceed unity.

For ease of reference, the complete listing of the model equations, objective function, stopping rule and initial values is given in Appendix A.

IV. AN EXAMPLE OF USE OF THE MODEL

A. EXAMPLE PROBLEM

The purpose of this thesis is to formulate a mathematical model of combat in which supply and logistics considerations are explicitly incorporated. The importance of supplies to the outcome of a battle or campaign has been discussed previously. This model can be used to explore various tactics and command decisions in situations where supplies and resupply capacity, as well as reserve forces and reinforcement rates, are of concern to the decision maker. The model is extremely flexible; it can be modified, expanded, or simplified to fit a great many situations.

The general model formulated above is a differential game where both commanders are free to make decisions. There appears to be little hope of developing analytic results in this general two-sided case. In order to develop a feel for the consequences of such a model a simplified version was considered. In this special case, optimization of the combat dynamics through application of optimal control theory has been considered by LCDR R. Powers at the Naval Postgraduate School. This special case was considered in order to provide Powers with inputs for the determination of the optimal policy from among extremal¹ candidates.

¹An extremal is a trajectory on which the necessary conditions of optimality are satisfied everywhere in time.

The following scenario describes the problem of interest. Force X has a main battle force and a reserve force. Force Y has only a main battle force. Force X has a logistics pipeline Force Y does not. Force X used only one group of supplies, which are used only by troops in combat. Figure 3 depicts the situation. Note that no pipeline defense or attack forces are employed.

The problem may be stated in mathematical terms as:

$$\text{MAX } (M1X1 (T) + M2X2 (T) - N1Y1 (T))$$

$$\text{rxl}$$

Subject to:

$$\frac{dX1(t)}{dt} = -aY(t) + rxl$$

$$\frac{dX2(t)}{dt} = -rxl$$

$$\frac{dY(t)}{dt} = -b X1(t)$$

$$\frac{dS(t)}{dt} = -CX1(t) + P(t)$$

with stopping rule:

$$X_1(t) = 0, Y2(T) = 0, S(T) < 0.$$

The following points should be noted in this submodel: all pipeline capacity is allocated to combat supplies and the attrition-rate coefficients are taken to be constants. The importance of supply constraints appears in the stopping rule ($S(T) < 0$).

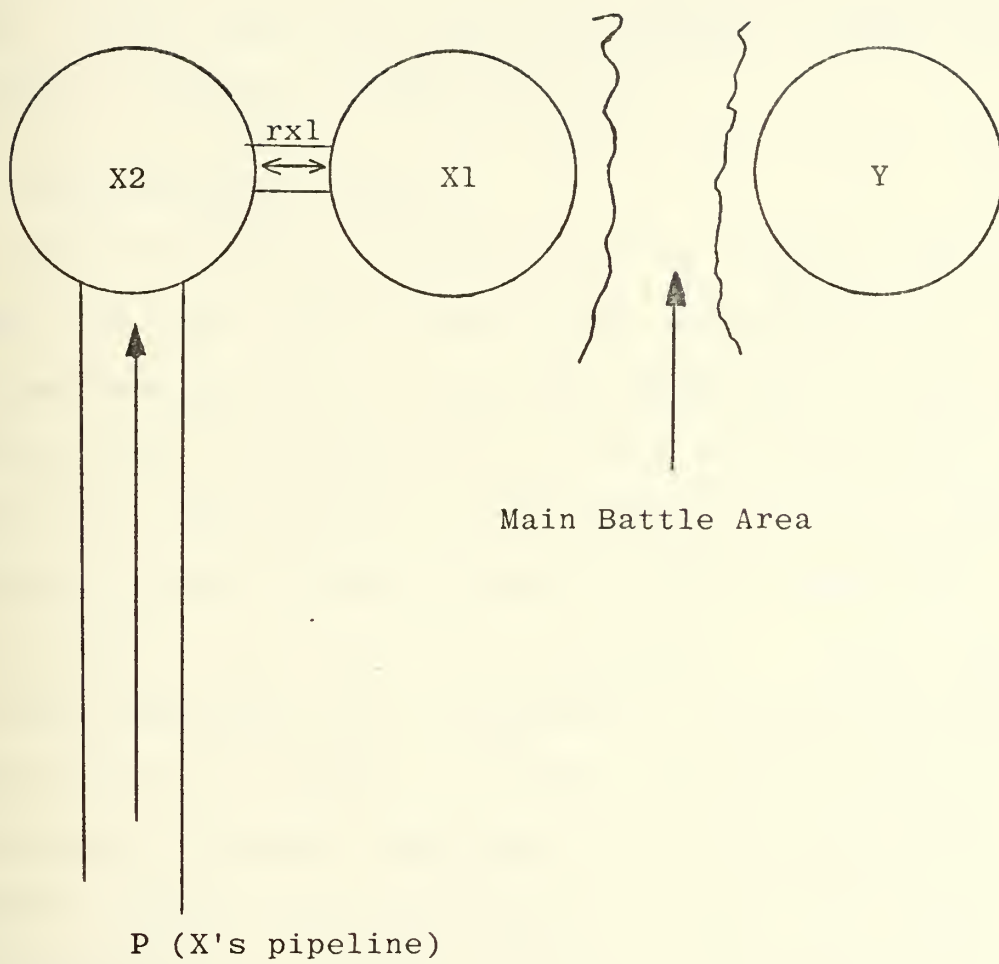


Figure 3

As stated above, optimal tactics for X have been studied by LCDR Robert Powers of Naval Postgraduate School via modern optimal control theory. The author developed a digital computer program to compute the value of the criterion functional (i.e., objective function) for various extremal policies developed in Powers' research. (The determination of an optimal policy here follows the general method outlined by Taylor in [Ref. 10] (see also [Ref. 11]). With initial values of X_1 , X_2 , S , a , b , and c given, trajectories of the X and Y force levels over time were computed. These trajectories are presented in Figures 4 and 5.

B. DISCUSSION OF RESULTS

From the curves in Figure 4, it is apparent that in the case where $rx_1 = +10.0$, the X force annihilates the Y force. In the case where $rx_1 = -10.0$, X_1 force goes to zero, but the X_2 force increases to 111. The Y force decreases from 50 to 33. Which value of rx_1 is optimal depends on the values assigned to the survivors. For the case where $M_1 = M_2 = N = 1.0$, $rx_1 = 10.0$ is optimal. In this case the X force commander should immediately reinforce his main battle force to achieve a victory, provided supplies do not become a binding constraint. In the problem above, the initial supply level was chosen so as not to be constraining, by determining the length of battle, and choosing initial supply level greater than the consumption rate times the initial force level times the length of the battle. If supplies become a binding constraint, it appears that the X

rx1 = +10.0 $\frac{\text{men}}{\text{min}}$

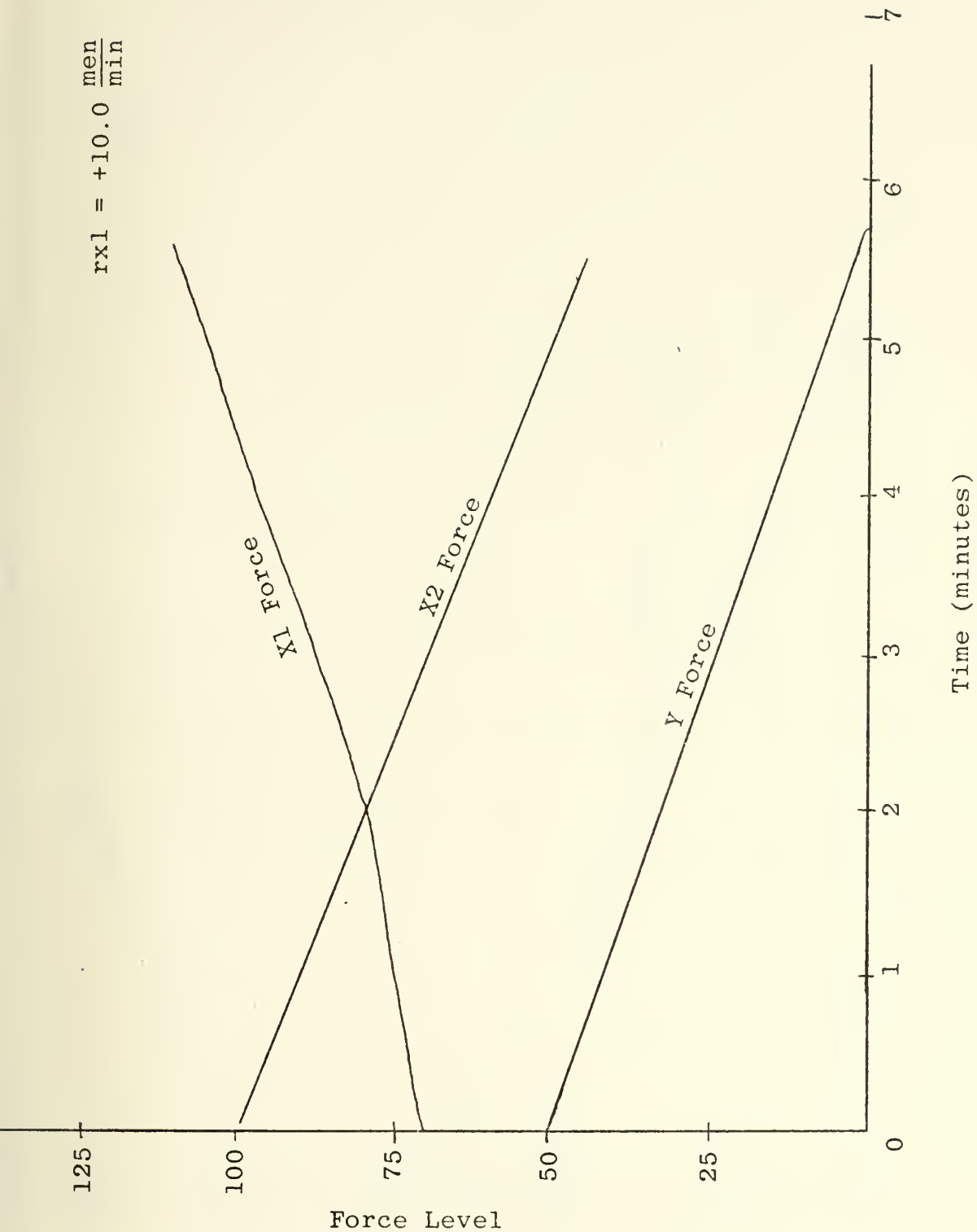


Figure 4

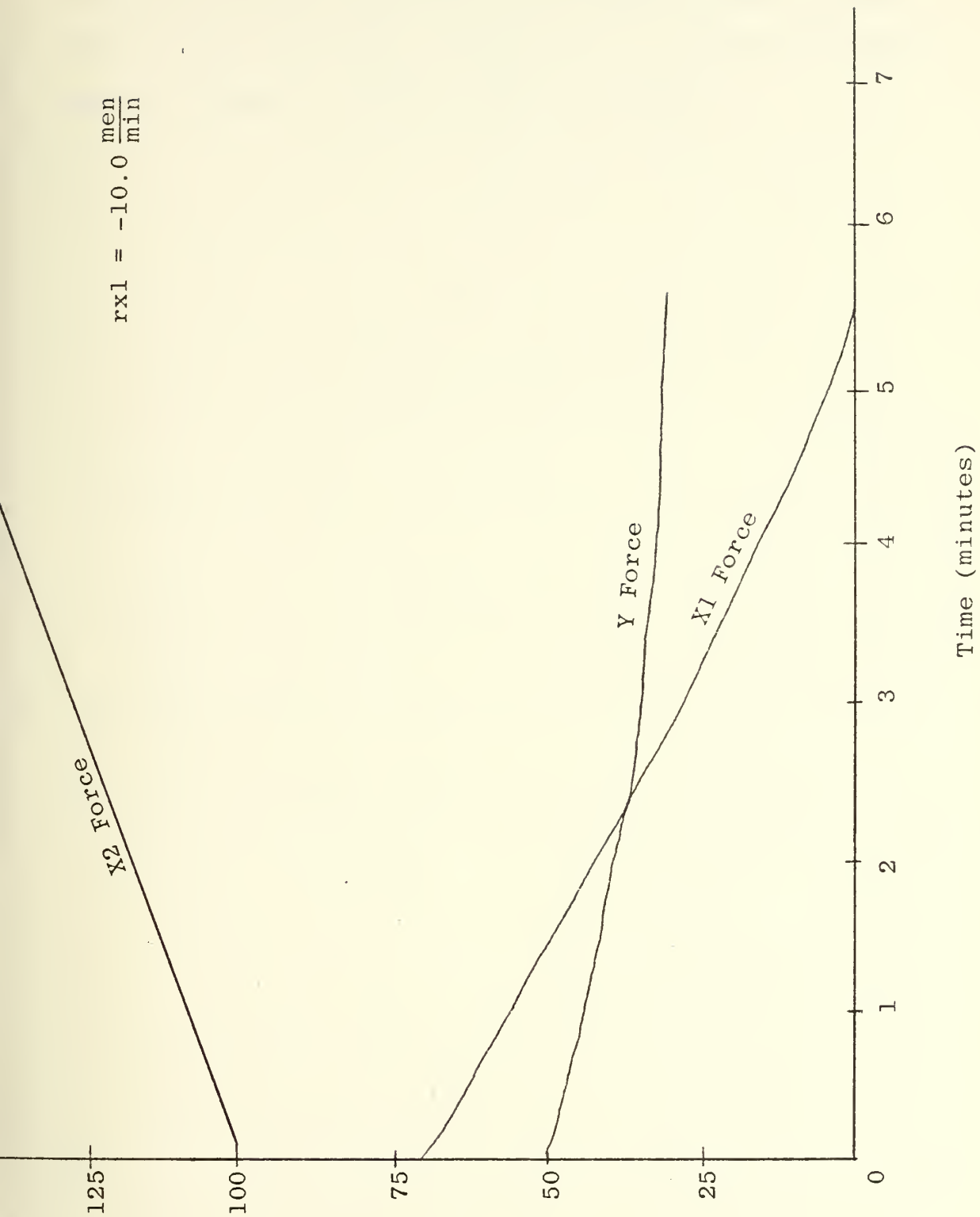


FIGURE 5

force commander should commit as many forces as possible to the main battle force until the supply level becomes close to zero, and then maintain X_1 at a level such that supply consumption equals rate of resupply.

V. ANALYSIS OF THE MODEL USING

"STEADY STATE" CONDITIONS

An analytical approach which can be used to study this model is taken from a Naval Ordnance Laboratory analytical model of a mining campaign [Ref. 12]. This approach, which we will call "steady state" analysis, results from imagining the following state of affairs in the model scenario: both the X and Y force commanders find that a stalemate exists. Neither X nor Y is winning the main battle, and neither side is building up or running down supply levels. Both sides have no reserve forces (or perhaps reinforcements from outside the model) sufficiently adequate to compensate for combat attrition which is taking place.

When this "steady state" condition exists, all time derivatives are zero, and quantities which are indicated as functions of time are constants. The mathematics is now simplified significantly. In order to visualize this method of analysis, consider the following simplified example.

$$\text{Let } \frac{dX_1(t)}{dt} = -aY_1(t) - rx_1 = 0$$

$$\text{and } \frac{dY_1(t)}{dt} = -bX_1(t) + ry_1 = 0$$

$$\text{Then } rx_1 = aY_1(t) \text{ and } ry_1 = bX_1(t)$$

Let $R = \frac{ry_1}{rx_1}$ be a figure of merit, which has the property that it is a single number which increases as the situation becomes more favorable to X, and decreases as the situation

becomes more favorable to Y. In this example, if rx_1 , the rate at which X reinforces X_1 , decreases the ratio $\frac{ry_1}{rx_1}$ increases and thus R increases. Intuitively, we would expect that if X could increase his combat effectiveness against Y, i.e. increase the attrition-rate coefficient b, it would be favorable to X. To see that our intuition is correct, write

$$R = \frac{ry_1}{rx_1} = \frac{bX_1(t)}{aY_1(t)}$$

If we consider b as being a variable, and take the partial derivative of R with respect to b, we obtain $\frac{\partial R}{\partial b} = \frac{X_1(t)}{aY_1(t)}$. Since $a, X_1(t)$ and $Y_1(t)$ are all non-negative numbers, we see that $\frac{\partial R}{\partial b} > 0$ which means that an increase in b will result in an increase in R, as expected.

To apply this analysis to the general model developed in this thesis, we must consider that X_2 and Y_2 , the reserve forces, are "outside the model" because if any of the combat forces are being reinforced (as they must be if they are taking casualties and the force levels remain constant, or in "steady state") obviously $\frac{dX_2(t)}{dt}$ and $\frac{dY_2(t)}{dt}$ cannot be zero. With this minor modification, we can set all other time derivatives equal to zero. For a figure of merit, let us define R as before, in terms of reinforcement rates. The total reinforcement rate for Y is $ry_1 + ry_3 + ry_4 = ry$; similarly for X. Thus $R = \frac{ry}{rx} = \frac{ry_1 + ry_3 + ry_4}{rx_1 + rx_3 + rx_4}$. We see that anything which reduces the X reinforcement rate, or increases the Y reinforcement rate, increases R and is more

favorable to X. Note that R can be interpreted as the casualty exchange ratio, since in steady state all casualties are replaced, and thus $R = \frac{\text{casualty rate of Y}}{\text{casualty rate of X}}$.

From the combat dynamics equations (see Appendix A), with time derivatives set equal to zero, we obtain:

$$rx1 = (1-e^{-S_{ya}(t)})(1-e^{-S_{yf}(t)})Y1(t)$$

$$rx3 = (1-e^{-S_{ya}(t)})(1-e^{-S_{yf}(t)})Y3(t) \text{ and similarly for}$$

rx4, ry1, ry3, ry4. From these equations, one obtains:

$$R = \frac{(1-e^{-S_{xa}(t)})(1-e^{-S_{xf}(t)})(X1(t) + X3(t) + X4(t))}{(1-e^{-S_{ya}(t)})(1-e^{-S_{yf}(t)})(Y1(t) + Y3(t) + Y4(t))}$$

Suppose that the X commander now has an opportunity to increase his pipeline capacity by using some of his present capacity to obtain some capital goods. He is presently using his pipeline for only food and ammo, and thus must take some capacity away from one supply group or the other (or possibly both). Which group should provide the capacity for capital goods? One way to approach this problem is to determine the change on R with a change in both S_{xa} and S_{xf} , by taking partial derivatives of R with respect to S_{xa} and S_{xf} . In this example, from the symmetry of the terms $(1-e^{-S_{xa}(t)})$ and $(1-e^{-S_{xf}(t)})$ we see by inspection that these partial

derivatives will be equal with equal values of S_{xa} and S_{xf} . Recall from the discussion of the construction of these attrition rate coefficients, however, that there was a shape parameter, α , which we had assumed to be numerically equal to one. With a different shape parameter for food and for ammo, then, the partial derivatives will have different values. After taking the partial derivatives and evaluating each of them using the existing supply and force levels, a comparison is made between them to see which is smaller. Since supplies are being taken away, and this will obviously decrease R , the supply group which decreases R by the least amount should be chosen.

In this manner, the X commander will temporarily decrease his combat effectiveness and increase his casualty rate, but in the least harmful way, in order to increase his pipeline capacity. After his capital goods have arrived and are installed to increase his pipeline capacity, he performs a similar analysis to determine whether to use his additional capacity for food or ammo or both.

The X commander must be careful in this type of analysis and decision process, however. By temporarily decreasing his resupply rate of ammo, for example, he is taking the model out of the steady state condition. He will be using ammo supplies faster than they will be replaced and he must be sure to have sufficient stock on hand to cover this temporary shortage. In addition, as lower and lower levels of ammo are reached, the partial derivatives must be

re-evaluated and compared at these new supply levels, in order to see if the change in R is less with respect to a change in ammo than it is with respect to a change in food. If it is not, the commander will want to allocate more of the existing pipeline capacity to ammo and less to food, until $\frac{\partial R}{\partial S_{xa}} = \frac{\partial R}{\partial S_{xf}}$, which implies an optimal allocation. (For a short discussion on optimal allocation, see Ref. 13)

The above example is presented as a sample of the type of insight which can be obtained using this model and steady state analysis techniques. For a more detailed description of the application of steady state analysis to a Lanchester-type model, the reader is referred to Ref. 12.

VI. CONCLUSIONS

The purpose of this thesis was to formulate a mathematical model of combat in which logistics were taken into consideration. The model developed is broad enough in scope to be applicable in a large number of situations. While a closed form analytical "solution" to the model may not be found, numerical results via finite difference methods may be generated using a digital computer. Using this method, one may run the model with various combinations of decision variable values, and by comparing the results, gain insight into which decisions are better. In certain simplified cases, such as the example provided, an optimal strategy may be determined using the methodology of optimal control theory. Finally, insight into a combat situation may be gained using the "steady state" analysis techniques previously discussed. Although the steady state analysis will not yield the time relationships of the variables in the model, much can be learned in a qualitative way about the nature of a combat situation in which logistics must be considered, and optimal allocation of pipeline capacity may be determined. Also, from the steady state analysis, one may gain insight into the trade-off between casualty rates and supply levels, by evaluating the partial derivatives of R (casualty exchange ratio) with respect to ammo and food supply levels.

VII. NEED FOR FURTHER RESEARCH

It was previously mentioned in Section II that the actual functional form of the logistics dependent combat effectiveness, or attrition-rate coefficient, is an operations research problem. Construction of these functions, perhaps from historical data, would make this model more applicable to real world situations.

We have seen an example of solution of a much-simplified submodel using optimal control theory.

As more research is done in this field of mathematics, more generalized versions of this model will perhaps be solvable for an optimal strategy.

An immediate follow-up to this formulation would be the writing of a digital computer program for the general model, as contained in Appendix A, for use in comparing various tactics and decisions for a particular scenario. Such a program might be of value as a teaching aid for future students of combat modelling.

APPENDIX A

MODEL SUMMARY

This appendix contains the mathematical description of the Lanchester-type model of combat with logistics considerations. For derivations and explanation of equations, refer to Section III.

For the X commander, the objective is to:

$$\begin{array}{ll} \text{Maximize} & (M_1 X_1(T) + M_2 X_2(T) - N_1 Y_1(T) - N_2 Y_2(T)); \quad i = 1, 3, 4; \\ r_{xi}, \rho_{xj} & j = a, f, c \end{array}$$

Subject to:

$$\frac{dX_1(t)}{dt} = - (1 - e^{-S_{ya}(t)})(1 - e^{-S_{yf}(t)}) Y_1(t) + r_{x1}(t)$$

$$\frac{dY_1(t)}{dt} = - (1 - e^{-S_{xa}(t)})(1 - e^{-S_{xf}(t)}) X_1(t) + r_{y1}(t)$$

$$\begin{aligned} \frac{dX_2(t)}{dt} = & -r_{x1}(t) - r_{x3}(t) - r_{x4}(t) \quad (\text{if } X_2 = 0, \quad r_{x1} + r_{x3} + r_{x4} \\ & \leq 0) \end{aligned}$$

$$\begin{aligned} \frac{dY_2(t)}{dt} = & -r_{y1}(t) - r_{y3}(t) - r_{y4}(t) \quad (\text{if } Y_2 = 0, \quad r_{y1} + r_{y3} + r_{y4} \\ & \leq 0) \end{aligned}$$

$$\frac{dX_3(t)}{dt} = - (1 - e^{-S_{ya}(t)})(1 - e^{-S_{yf}(t)}) Y_4(t) + r_{x3}(t) \quad (X_3 > 0)$$

$$\frac{dY_3(t)}{dt} = - (1-e^{-S_{xa}(t)})(1-e^{-S_{xf}(t)})X_4(t) + r_{y3}(t) \quad (Y_3 > 0)$$

$$\frac{dX_4(t)}{dt} = - (1-e^{-S_{ya}(t)})(1-e^{-S_{xf}(t)})Y_3(t) + r_{x4}(t) \quad (X_4 > 0)$$

$$\frac{dY_4(t)}{dt} = - (1-e^{-S_{xa}(t)})(1-e^{-S_{xf}(t)})X_3(t) + r_{y4}(t) \quad (Y_4 > 0)$$

$$\frac{dP(t)}{dt} = -a_y(Y_4(t) - X_3(t)) + \rho_{xc}P(t) \quad (\text{for } Y_4(t) \geq X_3(t))$$

$$\frac{dP(t)}{dt} = \rho_{xc}P(t) \quad (\text{for } Y_4(t) < X_3(t))$$

$$\frac{dQ(t)}{dt} = -a_x(X_4(t) - Y_3(t)) + \rho_{xc}Q(t) \quad (\text{for } X_4(t) \geq Y_3(t))$$

$$\frac{dQ(t)}{dt} = \rho_{yc}Q(t) \quad (\text{for } X_4(t) < Y_3(t))$$

$$\frac{dS_{xa}(t)}{dt} = -C_{xa}(X_1(t) + X_3(t) + X_4(t) + \rho_{xa}P(t))$$

$$\frac{dS_{ya}(t)}{dt} = -C_{ya}(Y_1(t) + Y_3(t) + Y_4(t)) + \rho_{ya}Q(t)$$

$$\frac{dS_{xf}(t)}{dt} = -C_{xf}(X(t)) + \rho_{xf}P(t)$$

$$\frac{dS_{yf}(t)}{dt} = -C_{yf}(Y(t)) + \rho_{yf}Q(t)$$

$$X(t) = X_1(t) + X_2(t) + X_3(t) + X_4(t)$$

$$Y(t) = Y_1(t) + Y_2(t) + Y_3(t) + Y_4(t)$$

Initial Values:

$$X_1^\circ, X_2^\circ, X_3^\circ, X_4^\circ, P^\circ, Q^\circ, Y_1^\circ, Y_2^\circ, Y_3^\circ, Y_4^\circ,$$

$$S_{xa}^\circ, S_{xf}^\circ, S_{ya}^\circ, S_{yf}^\circ$$

Input Parameters:

$$\rho_{ya}, \rho_{yf}, \rho_{yc}, r_{y1}, r_{y3}, r_{y4}, M_1, M_2, N_1, N_2$$

$$a_y, a_x, C_{xa}, C_{ya}, C_{xf}, C_{yf}$$

Decision Variable Constraints:

$$r_{x1} \min \leq r_{x1} \leq r_{x1} \max$$

$$r_{x3} \min \leq r_{x3} \leq r_{x3} \max$$

$$r_{x4} \min \leq r_{x4} \leq r_{x4} \max$$

$$\rho_{xa} \geq 0$$

$$\rho_{xf} \geq 0$$

$$\rho_{xc} \geq 0$$

$$\rho_{xa} + \rho_{xf} + \rho_{xc} \leq 1$$

Stopping Rule:

$X_1 = 0$, or $Y_1 = 0$, or $S_{xf} = 0$, or $S_{xa} = 0$, or $S_{xf} = 0$,
or $S_{yf} = 0$.

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which are used to increase or replace the capacity of the logistics pipeline. Lanchester attrition-rate coefficients are considered to be functions of the level of food and ammo supplies.

In the model, each opponent has a main battle force, a reserve force, a logistics pipeline defense force, and a force which may attack the other side's logistic pipeline. Differential equations for the combat dynamics are derived, and some possible objectives and battle termination conditions are suggested.

An example of use of the model is given, and some analytical techniques for studying the model are discussed. Related topics for further research are suggested.

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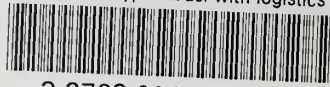
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